Spiral plat sans courbes terminales

Anisochronisme en position horizontale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times 10^{-3}\,mm^2 \quad TOL := 10^{-9}$

$d2_{sp} = 4.52\,mm \quad d1_{sp} = 1.1\,mm \quad p_{sp} = 0.135\,mm \quad n_{sp} = 12.667$

$L := L_{sp} \quad L = 11.182\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.56\times 10^3\,deg$

Position du piton $\quad r_P := 0.5\cdot d2_{sp} \quad \alpha_P := 0 \quad x_P := r_P\cdot\cos(\alpha_P) \quad y_P := r_P\cdot\sin(\alpha_P)$

$x_P = 2.26\,mm \quad y_P = 0\,mm \quad z_P := x_P + i\cdot y_P$

Position du point d'attache à la virole $\quad r_V := 0.5\cdot d1_{sp} \quad \alpha_V(\theta) := \psi_0 + \theta \quad x_V(\theta) := r_V\cdot\cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot\sin(\alpha_V(\theta))$

Forme naturelle du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_P - a\cdot\alpha \quad x_0(\alpha) := r_s(\alpha)\cdot\cos(\alpha) \quad y_0(\alpha) := r_s(\alpha)\cdot\sin(\alpha) \quad z_0(\alpha) := r_s(\alpha)\cdot\exp(i\cdot\alpha)$

$r'(\alpha) := \dfrac{d}{d\alpha}r_s(\alpha) \quad s(\alpha) := \dfrac{\pi}{p_{sp}}\cdot\left(r_P{}^2 - r_s(\alpha)^2\right) \quad s(\alpha) := r_P\cdot\alpha - \dfrac{a}{2}\cdot\alpha^2 \quad s(\psi_0) = 11.182\,cm$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Déplacement de la virole libre

$$\sigma 2 := \frac{1}{L}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot\overline{z_0(\alpha)}\cdot r_s(\alpha)\,d\alpha \qquad \sigma 2 = 2.705\,mm^2$$

$$\Delta_1(\theta) := \frac{i\cdot\theta}{L}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot\exp\left(i\cdot\theta\cdot\frac{s(\alpha)}{L}\right)\cdot r_s(\alpha)\,d\alpha$$

$$\Delta_1(\theta) := \frac{i\cdot\theta}{L}\cdot\int_0^{\psi_0} \exp\left[i\cdot\alpha\cdot\left[1+\frac{\theta}{L}\cdot\left(r_P - \frac{a}{2}\cdot\alpha\right)\right]\right]\cdot(r_P - a\cdot\alpha)^2\,d\alpha$$

$u_1(\theta) := Re(\Delta_1(\theta)) \quad v_1(\theta) := Im(\Delta_1(\theta)) \quad u_1(\theta_0) = -0.207\,mm \quad v_1(\theta_0) = 0.01\,mm$

Perturbation de période - spiral non déformé en position de repos

$X(\theta) := \frac{\left(\left|\Delta_1(\theta)\right|\right)^2}{\sigma 2}$ $\quad \theta(\varphi) := \theta_0 \cdot \cos(\varphi)$ $\quad \gamma(\theta) := \frac{d}{d\theta} X(\theta)$

$Delta(\theta_0) := \frac{-1}{2 \cdot \pi \cdot \theta_0} \cdot \int_0^{2 \cdot \pi} \gamma(\theta_0 \cdot \cos(\varphi)) \cdot \cos(\varphi) \, d\varphi$ $\quad Delta(\theta_0) = -8.459 \times 10^{-4}$

$\mu(\theta_0) := -86400 \cdot Delta(\theta_0)$ $\quad \boxed{\mu(\theta_0) = 73.086}$ $\quad \boxed{\mu(220 \cdot deg) = 65.987}$ $\quad \boxed{\mu(300 \cdot deg) = 77.069}$

Approximations de Haag

$\mathbf{OP} := r_P$ $\quad \mathbf{OV} := r_V \cdot e^{i \cdot \psi_0}$ $\quad \mathbf{w}(\theta) := \frac{-\theta}{L} \cdot \left[\left(r_P - 2 \cdot i \cdot a - \frac{\theta}{L} \cdot r_P^2 \right) \cdot \mathbf{OP} - \left(r_V - 2 \cdot i \cdot a - \frac{\theta}{L} \cdot r_V^2 \right) \cdot \mathbf{OV} \cdot e^{i \cdot \theta} \right]$

$X_w(\theta) := \frac{\left(\left|\mathbf{w}(\theta)\right|\right)^2}{\sigma 2}$ $\quad \gamma_w(\theta) := \frac{d}{d\theta} X_w(\theta)$ $\quad \delta_w(\theta_0) := \frac{-1}{2 \cdot \pi \cdot \theta_0} \cdot \int_0^{2 \cdot \pi} \gamma_w(\theta_0 \cdot \cos(\varphi)) \cdot \cos(\varphi) \, d\varphi$

$\mu_w(\theta_0) := -86400 \cdot \delta_w(\theta_0)$ $\quad \boxed{\mu_w(\theta_0) = 71.238}$ $\quad \boxed{\mu_w(220 \cdot deg) = 64.809}$ $\quad \boxed{\mu_w(300 \cdot deg) = 74.727}$

$F(\theta_0) := J0(\theta_0) - \theta_0 \cdot J1(\theta_0)$ $\quad \delta_{ah}(\theta_0) := \frac{2}{L^2} \cdot \frac{1}{r_P^2 + r_V^2} \cdot \left[-\left(r_P^4 + r_V^4 \right) + 2 \cdot r_P^2 \cdot r_V^2 \cdot F(\theta_0) \cdot \cos(\psi_0) \right]$

$\mu_{ah}(\theta_0) := -86400 \cdot \delta_{ah}(\theta_0)$ $\quad \boxed{\mu_{ah}(\theta_0) = 71.063}$ $\quad \boxed{\mu_{ah}(220 \cdot deg) = 65.333}$ $\quad \boxed{\mu_{ah}(300 \cdot deg) = 73.609}$

$x := 100 \cdot deg$ $\quad \theta_{01} := racine(F(x), x)$ $\quad \theta_{01} = 71.951\, deg$ $\quad \theta_{m1} := racine\left(\frac{d}{dx} F(x), x\right)$ $\quad \theta_{m1} = 156.682\, deg$

$x := 300 \cdot deg$ $\quad \theta_{02} := racine(F(x), x)$ $\quad \theta_{02} = 233.737\, deg$ $\quad \theta_{m2} := racine\left(\frac{d}{dx} F(x), x\right)$ $\quad \theta_{m2} = 326.093\, deg$

$\theta_m := 100 \cdot deg, 110 \cdot deg .. 350 \cdot deg$

$\mu_{m1} := -86400 \cdot \delta_{ah}(\theta_{m1})$ $\quad \mu_{m1} = 61.602$ $\quad \mu_{m2} := -86400 \cdot \delta_{ah}(\theta_{m2})$ $\quad \mu_{m2} = 74.403$